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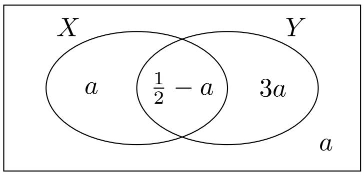
101. The parabola $y = ax^2 + 2x + 1$ passes through the point $(-2, 17)$. Find a.

102. Show that $0.\dot{4}\dot{2} = \frac{14}{33}$.

103. The probabilities of events X and Y are given in terms of $a \in \mathbb{R}$ as follows:



Find a.

104. Determine whether the point $(2, 3)$ lies inside, on, or outside the circle $x^2 + y^2 = 10$.

105. An expression is given as $p^{\log_p 3} \times q^{2 \log_q 2}$.

(a) Write down the law of indices that justifies the following simplification

$$q^{2 \log_q 2} \equiv \left(q^{\log_q 2}\right)^2.$$

(b) Hence, evaluate the expression.

106. Show that $\mathbf{r} = -\frac{7}{25}\mathbf{i} + \frac{24}{25}\mathbf{j}$ is a unit vector.

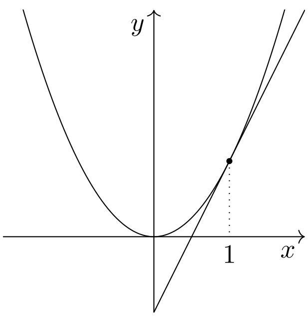
107. A projectile is launched from ground level over flat ground. The components of its initial velocity are 12.5 m/s horizontally and 19.6 m/s vertically.

(a) Write down the two main assumptions of the projectile model, re size and acceleration.

(b) Using $s = ut + \frac{1}{2}at^2$ for the vertical motion, show that the projectile lands after 4 seconds.

(c) Hence, show that the range is 50 metres.

108. The curve $y = x^2$ has a tangent drawn to it at $x = 1$.



By differentiating, show that this tangent has equation $y = 2x - 1$.

109. If $y = 2^x$, write 8^x in terms of y.

110. The small-angle approximation for $\sin\theta$, for θ in radians, is $\sin\theta \approx \theta$. Find the percentage error in this approximation at

(a) $\theta = 0.1$ radians,

(b) $\theta = 0.5$ radians.

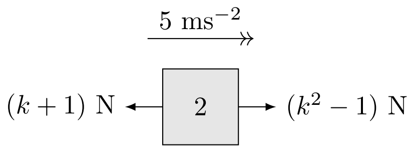
111. In the following equation, a is a constant:

$$(3x - 1)(2x + a)(x + a) = 0.$$

The equation has solution $x \in \{3, 6\}$. Find a.

112. By setting up a boundary equation $\mathrm{f}(x) = 0$ and sketching the graph $y = \mathrm{f}(x)$, solve the inequality $x^2 - 5x < 0$, giving your answer in set notation.

113. Forces act on an object as depicted:



Solve to find the positive value of k.

114. Find $\displaystyle\int 6x^2 + 5\,dx$.

115. The parabola $y = ax^2 + bx + c$ and the line $y = k$ intersect twice. At the points of intersection, the parabola has gradients m_1 and m_2. Prove that $m_1 + m_2 = 0$.

116. A *perfect number* is a positive integer which is equal to the sum of all its positive divisors except itself. So 6 is perfect, because $6 = 1 + 2 + 3$. Show that 28 is a perfect number.

117. Giving your answer in radians, find the remaining angle θ in a pentagon with interior angles

$$\left\{\tfrac{4\pi}{10}, \tfrac{5\pi}{10}, \tfrac{6\pi}{10}, \tfrac{7\pi}{10}, \theta\right\}.$$

118. Explain whether the following implications hold:

(a) $y = x^2 \implies x = y^{\frac{1}{2}}$,

(b) $y = x^3 \implies x = y^{\frac{1}{3}}$.

119. The lines $x + y = 1$, $2x - 5y = 9$ and $2x - y = k$ are concurrent. Find the value of the constant k.

120. By factorising, solve the following quadratic in 3^x:

$$\left(3^x\right)^2 - 6 \cdot \left(3^x\right) - 27 = 0.$$

121. Find the intersections of the following curves:

$$y = 5x^2 - 20x + 40,$$
$$y = 2x^2 + 15x + 52.$$

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122. Prove the following identity:

$$\frac{1}{b(ab-1)} + \frac{1}{b} \equiv \frac{a}{ab-1}.$$

123. An equation is given as

$$x^2 + y^2 = (x-1)^2 + (y-1)^2.$$

(a) Show algebraically that this equation defines a straight line.

(b) Using an argument based on distances between points, explain how you know that this straight line is the perpendicular bisector of the points $(0,0)$ and $(1,1)$.

124. Show that, if the relationship between x and y is quadratic, of the form $y = ax^2 + bx$, then y/x and x are related linearly.

125. A rectangle, sides length x and y, has perimeter 22 cm and area 28 cm^2. Find the lengths of its edges.

126. Four cards are chosen from a standard deck. State, with a reason, whether getting four of a kind is more probable if the cards are chosen

① with replacement,

② without replacement.

127. Show that the following expression is a quadratic in x with integer coefficients:

$$\left(x - 2 + \sqrt{11}\right)\left(x - 2 - \sqrt{11}\right).$$

128. A function is defined over the real numbers by the instruction $\text{f}(x) = \frac{1}{2}x^2 + 1$.

(a) Show that $\text{f}(x) = x$ has no real roots.

(b) Sketch the graphs $y = \text{f}(x)$ and $y = x$ on the same set of axes.

(c) Hence, show that the outputs of the function $\text{f}(x) = \frac{1}{2}x^2 + 1$ always exceed its inputs.

129. Prove that the product of two odd numbers is odd.

130. You are given that the straight lines $ax + y = 1$ and $11 - 2x + 3y = 0$ intersect at the point $(1, b)$. Find the constants a and b.

131. If $3y - 1 = 2x^{\frac{3}{2}}$, show that $\dfrac{dy}{dx} = \sqrt{x}$.

132. The quadratic $x^2 + 5x + k$ has $(x+2)$ as a factor. Find the value of k.

133. Write down the roots of the equation

$$\frac{(x+1)(x-1)}{(x+3)(x-3)} = 0.$$

134. One of the following statements is true; the other is not. Identify and disprove the false statement.

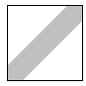
① $x^2 = 1 \implies x^5 = 1$,

② $x^5 = 1 \implies x^2 = 1$.

135. By setting up and solving a boundary equation, determine the integer value of x for which

$$3x^2 + 6x + 1 < 0.$$

136. A set of tiles has the following pattern on each.



Four such tiles are laid together, in a 2 by 2 grid. Find the probability that

(a) the four stripes form a square,

(b) the four stripes are all parallel.

137. Find the gradient of $y = x^5 - 4x^3 + 10x^2 + 5$ at the point with x coordinate 2.

138. Ten books are placed on a shelf. The two heaviest are chosen to go at the ends. Show that there are 80640 ways of arranging the books.

139. State, giving a reason, which of the implications $\implies$, $\impliedby$, $\iff$ links the following statements concerning a real number x:

① $(x-1)(x-2) = 0$,

② $x = 1$.

140. Find the distance between the points (a, b) and $(a+3, b-4)$, where a, b are real constants.

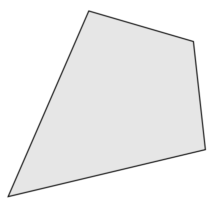
141. A woman of mass 55 kg is standing in a lift, which is accelerating upwards at 2 ms^{-2}.

(a) Draw a force diagram for the woman.

(b) Find the magnitude of the force exerted by the lift floor on her feet.

(c) Write down the magnitude of the force exerted by the woman on the lift floor.

142. A kite has diagonals of length 6 and 8.



Find the area of the kite.

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143. The quadratic equation $x^2+ax-6=0$ has distinct roots at $x=1$ and $x=b$. Find a and b.

144. A French riddle goes: "A waterlily doubles in size every day. On day 30 it will cover the pond, killing everything else. The gardener decides to act when the pond is half covered. On which day is that?"

145. Solve the equation $\dfrac{x+1}{(x-1)^2}=1$.

146. (a) Carry out the following evaluation:

$$16x^2-44x+6\Big|_{x=\frac{1}{2}}$$

(b) Hence, show that $(2x-1)$ is not a factor of the expression $16x^2-44x+6$, giving the name of the theorem you use.

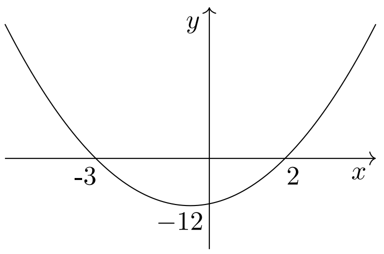
147. Show that $41x-5=100x^2$ has no real roots.

148. Find f' for the following functions:

(a) f defined by $f(x)=0$,
(b) f defined by $f(x)=1$,
(c) f defined by $f(x)=1+x$.

149. Solve $2\sin\theta=\sqrt{3}$, giving all values $\theta\in[0,360°)$.

150. Find the equation of the parabola shown below, on which the axes intercepts have been marked, giving your answer in expanded polynomial form.



151. Two angles of a triangle are 1.42 and 0.46 radians. Find the third, to 2dp.

152. Sketch the following graphs, where $0<a$,

(a) $y=x(x-a)$,
(b) $y=x(a-x)$,
(c) $y=x^2(x-a)$.

153. Solve the equation $3\left|10-3x\right|-12=0$.

154. A rectangular lawn with area 48 m^2 measures 10 metres diagonally. Find the perimeter of the lawn.

155. The numbers a,b,c,d are consecutive terms of an arithmetic progression. Prove that $a+d=b+c$.

156. Simplify the following sets:

(a) $(-\infty,1]\cup[-1,\infty)$,
(b) $(-\infty,-1]'\cap[1,\infty)'$.

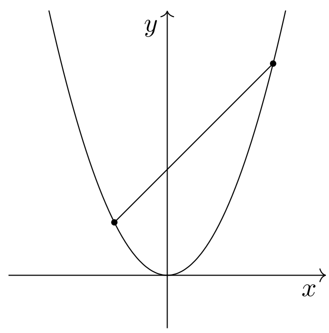
157. Three points A, B and C are given at coordinates $(3,0)$, $(4,-2)$ and $(1,-3)$. Determine which two of these points are closest to each other.

158. Find the probability that, if the letters of the word WORD are rearranged at random, they spell the original word.

159. By completing the square, or otherwise, show that, over the positive reals, the minimum value of the function $f(x)=x(x-5)$ is $-\frac{25}{4}$.

160. By factorising, solve $3x^4-14x^3+8x^2=0$.

161. A chord is drawn to the curve $y=x^2$ at the points where $x=-1$ and $x=2$.



Show that this chord crosses the y axis at $y=2$.

162. As x increases from zero, determine which of the following functions reaches an output of 100 first.

$$f(x)=4x+20,$$
$$g(x)=6x.$$

163. Simplify the following expressions:

(a) $\sqrt{2}+\sqrt{8}+\sqrt{32}$,
(b) $\dfrac{1}{1-\sqrt{a}}+\dfrac{1}{1+\sqrt{a}}$.

164. Determine all values $n\in\mathbb{N}$ satisfying ${}^n\mathrm{C}_2=15$.

165. In each part, find all possible functions f which have the given derivative:

(a) $f'(x)=0$,
(b) $f'(x)=3$,
(c) $f'(x)=2x+1$.

166. Solve the equation $\dfrac{x}{x+1}-\dfrac{x-1}{x}=\dfrac{1}{2}$.

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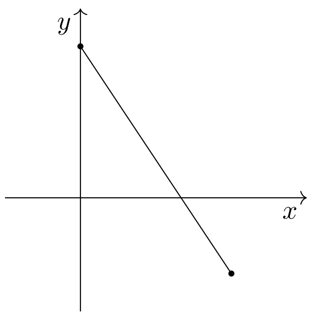
167. By sketching the boundary equation $x^2 + y^2 = 4$, shade the region of the (x, y) plane which satisfies the inequality $x^2 \leq 4 - y^2$.

168. A line segment is defined by $x = 2t$, $y = 2 - 3t$, where t is a parameter taking values $t \in [0, 1]$.



Show that this line segment is never further than $\sqrt{5}$ from the origin.

169. Three forces, with magnitudes 2, 4, 6 Newtons, act on an object of mass 10 kg, which is in equilibrium. The smallest force is then removed. Determ1ine the subsequent acceleration of the object.

170. Rationalise the denominator of $\dfrac{1}{5 - 2\sqrt{6}}$.

171. An equation is given as $4^x + 2^x - 6 = 0$.

(a) Write 4^x in terms of 2^x.

(b) Hence, factorise the LHS.

(c) Solve for x.

172. Using a Venn Diagram, or otherwise, determine whether $A' \cup B$ and $A \cap B'$ are mutually exclusive.

173. Find, in terms of a, the area of the region enclosed by the lines $y = 2x$, $y = -4x$, and $x = a$.

174. Consider the function $f(x) = x^3 + 6x^2 - x - 30$.

(a) Show that $(x - 2)$ is a factor.

(b) Express $f(x)$ as the product of three factors.

(c) Hence, solve $f(x) = 0$.

175. *Goldbach's conjecture* is one of the most famous conjectures (i.e. unproved results) in mathematics. Goldbach claimed that every even number greater than 2 can be written as the sum of two primes. Verify Goldbach's conjecture up to $n = 20$.

176. Show that the lines $ax + by = c_1$ and $bx - ay = c_2$ are perpendicular.

177. A linear function has $f(1) = f'(1) = 2$. Find $f(6)$.

178. The formula for the sum of the first n terms of an arithmetic progression is given by

$$S_n = \tfrac{1}{2}n\left(2a + (n-1)d\right).$$

Find the sum of the first n odd integers.

179. Sketch $y = \sqrt{x}$.

180. Two dice are rolled. State which, if either, of the following events has the greater probability:

① two sixes,
② a five and a six.

181. Determine whether the line $y = 2x - 1$ crosses the curve $y = x^2 - 2x + 3$.

182. True or false?

(a) $x^2 = 1 \iff x = \pm 1$,

(b) $x^3 = 1 \iff x = \pm 1$,

(c) $x^4 = 1 \iff x = \pm 1$.

183. A circle C_1 has equation $x^2 + x + y^2 + y = 0$.

(a) Find the centre and radius of C_1.

(b) Show that $y = -x$ is tangent to C_1.

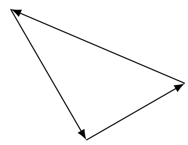
(c) Give the equation of C_2, the circle produced when C_1 is reflected in $y = -x$.

(d) Sketch both circles and the tangent line on the same set of axes.

184. Simplify $\dfrac{2\sqrt{2}}{4 - \sqrt{8}}$.

185. Three forces, with magnitudes 45, 60, and 75 N, act on an object, which remains in equilibrium.

(a) Explain how you know that the three forces, added together tip-to-tail as vectors, must form a closed triangle, as shown below:



(b) Find the angle between the 45 and 60 N forces.

186. The parabola $x = y^2 + 1$ and the line $y = mx + c$ intersect at $y = -2$ and $y = 3$.

(a) Set up simultaneous equations in m and c.

(b) Hence, find the equation of the line.

187. Find $\dfrac{dy}{dx}$ for the following graphs:

(a) $y = (x - 1)(x - 2)$,

(b) $y = (\sqrt{x} - 1)(x - 2)$.

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188. A cubic graph has equation $y = (x - a)(x - b)^2$, where $0 < a < b$. Sketch the graph, labelling the axis intercepts in terms of a and b.

189. Ten names are put into a hat, to be drawn without replacement. Find the probability that the first three names drawn out are the first three names alphabetically, in alphabetical order.

190. An irregular pentagon has sides defined, in order around the perimeter, by the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}, -\mathbf{a}, \mathbf{d}$. Show that $\mathbf{b} + \mathbf{c} + \mathbf{d} = 0$.

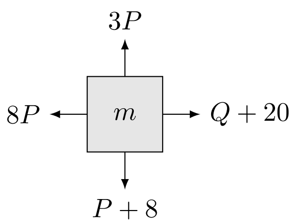
191. In each case, state, with a reason, whether the value of the limit is zero, one or infinity.

(a) $\displaystyle\lim_{x\to\infty} \frac{3^x}{2^x + 1}$,

(b) $\displaystyle\lim_{x\to\infty} \frac{3^x}{3^x + 1}$,

(c) $\displaystyle\lim_{x\to\infty} \frac{3^x}{4^x + 1}$.

192. Four forces keep an object in equilibrium:



Solve to find P and Q.

193. A rectangle has perimeter $8\sqrt{2}$ cm and area 6 cm^2. By setting up simultaneous equations and solving, show that the diagonals are $\sqrt{20}$ cm long.

194. Events A and B are independent, with $\mathbb{P}(A) = \frac{1}{3}$ and $\mathbb{P}(A \cap B) = \frac{1}{12}$. Find the following:

(a) $\mathbb{P}(B)$,

(b) $\mathbb{P}(A \cup B)$.

195. By locating a double root, show that $y = 2x - 1$ is a tangent to $y = x^2$.

196. Describe the quadrilaterals which have vertices at the following points:

(a) O, $(1, 0)$, $(2, 1)$, $(2, 2)$.

(b) O, $(2, -1)$, $(2, 1)$, $(3, 0)$.

197. A curve C has first derivative

$$\frac{dy}{dx} = 4x - 1.$$

C passes through $(0, 3)$. Find y in terms of x.

198. Determine the value of p and the value of q in the following identity:

$$1 + \frac{px}{1 - x} \equiv \frac{q}{1 - x}.$$

199. On the same axes, for positive constants a, b, sketch the graphs

(a) $\dfrac{y - b}{x - a} = 10$

(b) $\dfrac{y - b}{x - a} = \dfrac{1}{10}$.

200. One radian is defined as the angle subtended at the centre of a unit circle by an arc of unit length. Use this definition to prove that there are 2π radians in a full revolution.

——— End of 2nd Hundred ———

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